play a negligible rôle. In most texts in common use, the determinantal form for the area of a triangle is a solitary instance. Professor Foraker in the second discussion below gives an introduction to the use of determinants in the elementary field. Most of his work consists of the statement of formulae; one theorem is proved, relating to the collinearity of the circumcenter, centroid, and orthocenter of a triangle. It is scarcely to be expected that the material would be suitable for ordinary class use, but it will be of interest to a few students in any class.

The third discussion is a plea for the more general use of the history of mathematics in connection with secondary and collegiate instruction. Attracted by an announcement of the plans of the National Committee on Mathematical Requirements, Mr. Laurin Zilliacus of the Bedales School in Petersfield, England, has written a letter on the subject to Professor H. W. Tyler, a member of the committee. With the permission of the writer, the recipient, and Professor J. W. Young, chairman of the committee, we reproduce nearly the entire letter, retaining the personal form in which it is written. The fact that Mr. Zilliacus has actual experience of the growth of interest and enthusiasm in a class through the use of historical material is of more value than any amount of mere theorizing on the question.

Another article basing pedagogic recommendations on the result of personal experience is found in the last discussion, by Professor Ettlinger, on the use of graphical methods in trigonometry. It is nearly certain that all teachers make some use, in their teaching, of the ideas suggested by Professor Ettlinger; not all, however, have carried these ideas so far, or treated them with such emphasis. One incidental point in the paper moves the editor to a comment which he has long hoped to see made. It is pointed out that a correctly drawn figure is of great aid in attacking a problem. This is undeniably true in trigonometry, and especially in elementary geometry. Is it not also true, on the other hand, that sometimes a figure drawn with carefully planned *inaccuracy* is of extreme importance and inspiration?

I. Heats of Dilution.

By J. E. Trevor, Cornell University.

The "heats of dilution" of a solution are mathematical curiosities, in that these quantities are defined with reference to changes of the form of a function. When $\psi(x)$ is a given function, and δx is an arbitrary positive increment of the independent variable x, let it be supposed that a certain operation changes the value of a quantity φ from the value

$$\varphi_1 = \psi(x) + a \cdot \delta x$$

to the value

$$\varphi_2 = \psi(x + \delta x),$$

where a is a constant. The change of value $\varphi_2 - \varphi_1$ is due to the change of

form of the function $\varphi(x, \delta x)$, and the rate of change of φ per unit increment of x in $\psi(x)$ is

$$\lim_{\delta x \to 0} \frac{\psi(x + \delta x) - \psi(x) - a \cdot \delta x}{\delta x} = \frac{d(\psi - ax)}{dx}$$

The heats of dilution of a solution are defined by such limits.

Consider constant masses M_1, M_2 of two component substances, such as salt and water or water and alcohol, capable of forming a homogeneous liquid mixture. Let the state of the body constituted of the masses M_1, M_2 be such that any mass m_j of the jth component exists separately from the mixture of the masses $M_j - m_j$ and M_k, where M_k is the mass of the other component and $M_j \geq m_j \geq 0$. In the case $m_j = 0$, *i.e.*, when the body is a homogeneous "solution" σ in a state of stable thermodynamic equilibrium under the pressure p at the temperature θ, let $E(p, \theta, M_1, M_2)$ be the energy and $V(p, \theta, M_1, M_2)$ be the volume of the solution, where p, θ, M_1, M_2 are variable parameters. When the body is in a state σ_1 with separated parts, each in stable equilibrium at p, θ, its energy E_1 and volume V_1 are the sums of the energies and volumes of the parts. The "enthalpy" G of the solution σ and the enthalpy G_1 of the body in the state σ_1 are defined by the equations

$$G = E + pV, \qquad G_1 = E_1 + pV_1,$$

from which it follows that

$$E - E_1 = (G - G_1) - p(V - V_1).$$

When the separate parts are brought together, under the constant pressure p, the state σ_1 is transformed into the state σ and the work $-p(V - V_1)$ is absorbed by the body. Hence, by the energy law, the heat absorbed by the body is the quantity $G - G_1$. The particular case of this process realized when $m_j = M_j$ is the formation of the solution from its components. If g_j is the enthalpy of unit mass of the jth component, the enthalpy G_1 of the separate components is $M_1 g_1 + M_2 g_2$, and the "heat of mixing" ΔG of the solution is

$$\Delta G = G - M_1 g_1 - M_2 g_2, \tag{1}$$

where g_1, g_2 are functions of p, θ.

Consider now the dilution of the solution whose mass is $M_1 + M_2$, by addition of the mass δM_j of the jth component at p, θ. The value of the enthalpy of the body composed of the masses $M_1 + M_2$ and δM_j is

$$G(p, \theta, M_1, M_2) + g_j(p, \theta)\delta M_j \tag{2}$$

before the operation, and it is

$$G(p, \theta, M_j + \delta M_j, M_k) \tag{3}$$

after the operation. Both before and after the operation, the enthalpy of the body is a function of the constant quantities p, θ, M_j, M_k, δM_j. In the opera-

tion the enthalpy of the body changes in value because the form of the function changes from the form (2) to the form (3). Suppressing the constants p, θ, M_k, this change of value is

$$G(M_j + \delta M_j) - G(M_j) - g_j\delta M_j.$$

Since this expression denotes the heat absorbed by the body $(M_j,\ M_k,\ \delta M_j)$ during the dilution, we have that the heat absorbed per unit mass of diluent added is the limit

$$\lim_{\delta M_j \to 0} \frac{G(M_j + \delta M_j) - G(M_j) - g_j\delta M_j}{\delta M_j} = \frac{\partial G}{\partial M_j} - g_j.$$

This quantity is the "heat of dilution" Δ_j of the solution (M_1, M_2) for dilution by the jth component of the mixture.

Now by differentiating (1) we find

$$\frac{\partial \Delta G}{\partial M_j} = \frac{\partial G}{\partial M_j} - g_j.$$

Hence Δ_j is equal to the derivative $\partial \Delta G/\partial M_j$ of the heat of mixing. Further, since it is known that $G(p, \theta, M_1, M_2)$ is homogeneous of degree one in M_1, M_2, and hence by (1) that ΔG is homogeneous of degree one in these variables, we have

$$\Delta G = M_1\Delta_1 + M_2\Delta_2.$$

It thus appears that the heat of mixing of a solution is a linear function of the two heats of dilution.

II. Determinants in Elementary Analytic Geometry.

By F. A. Foraker, University of Pittsburgh.

The general purpose of this paper is to indicate how some results in elementary analytic geometry may be conveniently expressed in determinant form. We shall consistently use the notation

$$(1) \qquad \Delta = \begin{vmatrix} x_1, & y_1, & 1 \\ x_2, & y_2, & 1 \\ x_3, & y_3, & 1 \end{vmatrix}, \qquad D = \begin{vmatrix} a_1, & b_1, & c_1 \\ a_2, & b_2, & c_2 \\ a_3, & b_3, & c_3 \end{vmatrix};$$

and shall in the usual way indicate the cofactors of elements of determinants by corresponding capital letters. The points P_1, P_2, P_3 will be understood to have coördinates (x_1, y_1), (x_2, y_2), (x_3, y_3) respectively. It is well known that the area of the triangle $P_1P_2P_3$ is $\frac{1}{2}\Delta$, and that therefore the points P_1, P_2, P_3 are collinear if and only if $\Delta = 0$.

The equation of the line joining P_1 and P_2 is

$$(2) \qquad \begin{vmatrix} x, & y, & 1 \\ x_1, & y_1, & 1 \\ x_2, & y_2, & 1 \end{vmatrix} = 0;$$